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Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2015

Fifth Semester

Core Course—FUZZY MATHEMATICS

(For Model I B.Sc. Mathematics)

[2013 Admissions]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*
1 *mark each.*

1. Define partition of a set A.
2. State De Morgan's laws in crisp sets.
3. Give an example of a strong cutworthy property.
4. Define equilibrium points of a set A.
5. Define axiomatic skeleton of t-norms.
6. Define dual triple.
7. What is the value of $[0, 1] - [-6, 5]$?
8. Define a fuzzy number.
9. What do you mean by a quasi-contradiction ?
10. Define logic operations.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
2 *marks each.*

11. Define α-cut of a set A. Find the α-cut of a set A where $\alpha = 0.5$ and $A(x) = \frac{1}{1+10x}$.
12. Compute the scalar cardinality of $A = .4/v + .2/w + .5/x + .4/y + 1/z$.
13. Define the degree of sub-sethood of A in B.
14. State the second characterization theorem of fuzzy complements.

Turn over

15. If a complement c has an equilibrium e_c, then prove that ${}^{d}e_c = e_c$.

16. Define the bounded difference $i\,(a, b)$. Prove that it is a t-norm.

17. Show that $\{u_{\max}, i_{\min}, c(a) = 1 - a\}$ and satisfy De Morgan's law.

18. Let A and B are two fuzzy sets, whose membership functions are given by :

$A(x) = \frac{x}{x+1}$ and $B(x) = 1 - \frac{x}{10}$ for all $x \in \{0, 1, 2, \ldots, 10\} = X$. Calculate $S(|\bar{B}|, |\bar{A}|)$.

19. Show that $E(x) = \begin{cases} 1 & \text{for } x = 5 \\ 0 & \text{otherwise} \end{cases}$ is a fuzzy number.

20. Define a Boolean algebra.

21. What is quasi-tautology and quasi-contradiction ?

22. Define logic formulas.

$(8 \times 2 = 16)$

Part C (Descriptive/Short Essay Type Questions)

Answer any **six** *questions.*

4 marks each.

23. Let A, B be fuzzy sets defined on a universal set X. Prove that :

$|A| + |B| = |A \cup B| + |A \cap B|$, where $\cap, \cup$ are standard fuzzy intersection and union respectively.

24. Let A and B are fuzzy sets. Prove that $\alpha(\bar{A}) = {}^{(1-\alpha)+}\bar{A}$.

25. Let $A_i \in \mathcal{F}(X)$ for all $i \in I$ where I is an index set. Then prove that $\bigcup_{i \in I}{}^{\alpha+}A_i = {}^{\alpha+}\left(\bigcup_{i \in I} A_i\right)$ and

$\bigcap_{i \in I}{}^{\alpha+}A_i \subseteq {}^{\alpha+}\left(\bigcap_{i \in I} A_i\right)$. Also show that the inclusion cannot be replaced with equalities.

26. If C is a continuous fuzzy complement, then prove that C has a unique equilibrium.

27. Show that the standard fuzzy intersection is the only idempotent t-norm.

28. Show that the function :

$C(a)=\dfrac{\gamma^2(1-a)}{a+\gamma^2(1-a)}$ if $a\in[0,1],\gamma>0$ is a fuzzy complement. Plot the function for some values of γ. Find the generator of C.

29. Let A, B, E and F are closed intervals and if $A\subseteq E$ and $B\subseteq F$ then prove that :

(i) $A+B\subseteq E+F$.

(ii) $A-B\subseteq E-F$.

(iii) $A\cdot B\subseteq E\cdot F$.

(iv) $A/B\subseteq E/F \quad (0\notin F)$.

30. Explain Linguistic hedges.

31. Let sets of values X and Y be $X=\{x_1, x_2, x_3\}$ and $\{y_1, y_2\}$ respectively. Assume that a proposition "if X is A then Y is B" is given, where $A=\cdot6/x_1+1/x_2+\cdot9/x_3$ and $B=\cdot6/y_1+1/y_2$. Given the fact expressed by the proposition "X is A' " where $A'=\cdot5/x_1+\cdot9/x_2+1/x_3$. Then using the generalized modus ponens, derive a conclusion in the form "y is B".

$(6\times4=24)$

Part D (Long Essay)

Answer any **two** *questions.*
15 *marks each.*

32. State and prove second decomposition theorem of fuzzy sets.

33. Given an innovative fuzzy complement c and an increasing generator g of C, prove that the t-norm and t-conorm generated by g are dual with respect to c. Also show that the fuzzy operations i, u, c. Satisfy the law of excluded middle and the law of contradiction.

34. If $A(x)=\begin{cases}0 & \text{for } x\le-1 \text{ and } x>3\\ (x+1)/2 & \text{for } -1<x<1\\ (3+x)/2 & \text{for } 1<x<3\end{cases}$ and $B(x)=\begin{cases}0 & \text{for } x\le1 \text{ and } x>5\\ (x+1)/2 & \text{for } 1<x<3\\ (5-x)/2 & \text{for } 3<x\le5\end{cases}$

Then find A + B, A – B, A · B, A/B, B – A.

35. Explain the four types fuzzy propositions with their Canonical forms.

$(2\times15=30)$